

Parts I and III of the book are relevant to seventeenth century scholarship. Grosholz presents texts by Galileo, Newton, and Leibniz which display demonstrations employing some form of “highly structured ambiguity” that play a role in mathematical discovery and justification. For Grosholz, the epistemic virtues of fruitfulness and rigor of reasoning with signs are deeply intertwined. She calls this “productive ambiguity”. Given that the ambiguity at stake is “highly controlled” it excludes equivocation, a requirement Leibniz explicitly states. The use of ambiguous modes of representation in Galileo’s exposition of projectile motion is discussed in Part I. Galileo’s proof includes proportions, diagrams, numbers, and text in Latin that guides the reader. It is Galileo’s fruitful handling of “controlled ambiguity” that allows him to relate the finite with the infinitesimal: there is only one set of diagrams that must be read in two ways, as the accompanying text in Latin indicates - reading the depicted intervals as finite allows for the application of results of Euclid and Apollonius, reading the same intervals as infinitesimal “allows the diagram to stand as an analysis of accelerated motion” (p. 13).

Part III entitled “Geometry and Seventeenth Century Mechanics” is devoted to Descartes, Newton, and Leibniz. When Leibniz (“La Méthode de l’Universalité”, 1675) presents his “new instruments” of inquiry that would lead to the calculus he draws a distinction between uses of ambiguous signs that produce “equivocation” and uses of ambiguity that guarantee “univocity”. The first use must be eliminated but the ambiguity of the *letters* must be retained, as it is fruitful tool to allow for the “characteristic” to express the Principle of Continuity. As Grosholz notes, this guiding principle is at the basis of the Leibnizian idea that the finite, the infinitesimal, and the infinite are subject to the same rational constraints. To use *letters* ambiguously allows for the application of a “one universal rule that may embrace the infinitesimal, the finite, and the infinite”. The section on Leibniz on transcendental curves closes with Leibniz’s attempt during the 1690’s to link geometrical and dynamical aspects of the problem by using diagrams ambiguously. Grosholz shows how Leibniz exploits the “ambiguity” of diagrams that allow him to link the finite and the infinitesimal relating the geometrical and dynamical aspects in the context of his research on mathematics in relation to mechanics.

Grosholz claims that this is typical of reasoning in mathematics, even though this reasoning style has been mostly neglected by philosophers of mathematics. Such a reasoning style is clearly at odd with the stringent requirements of rigor that underlies the syntactic notion of proof that requires making explicit everything essential to deductive reasoning, so that “in any complete proof” of a theorem “figures” (diagrams) and other non syntactic forms of representation are “dispensable”: an idea that goes back to the German geometrician Pash (1882) who thought that this requirement was fulfilable.