Here are some comments for Karine Chemla, on the topic of generality, a propos her essay "Michel Chasles's Historiography of Geometry from the Viewpoint of the Value of Generality." A few suggestions and questions emerged from my discussion about it with Emiliano Ippoliti, a postdoctoral student from the University of Rome, who is here at Penn State this summer.

Projective geometry seems like a very promising mathematical area of research for exhibiting the value of generality, as well as a novel organization of a domain of research (resulting from generalization) that looks different from a deductive axiomatic structure. When you make this latter point, however, on p. 8, your claim is made in an understated way, at the end of a paragraph. "The organization of mathematical knowledge that inspires Chasles's comments connects propositions in a way different from what is the main concern in an axiomatic-deductive structure." This important point, which is showcased in *Les Neufs chapitres*, isn't brought forward clearly enough here, and perhaps ought to be articulated earlier in the essay.

Is it the case that Chasles produces a generalized object rather than abstract principles, and that this is what distinguishes his approach from the deductive axiomatic approach? The generalized object would be the gamut of geometrical objects that are related to each other through projection, that is, through transmutation, and which can therefore be treated uniformly (one proof fits all) by the methods Chasles develops.

Throughout your essay, in fact, many different things are noted (by Chasles and then by you) as general: first principles, propositions, methods, modalities of application of methods, proofs, terms and objects (the first more discursive and the second more thing-like), and properties. Does 'generality' mean the same thing in each instance? Perhaps this multiplicity should be noted more explicitly and up front. Perhaps 'generality' in relation to first principles in more Euclidean, and 'generality' in relation to objects is more Chasles-ean?

Throughout your essay, 'generality' is linked to a number of other terms (again, by Chasles and so by you): simplicity, economy, fruitfulness, connectedness, uniformity. This range of meanings produces another multiplicity, which might also be noted more explicitly and up front. One way to systematize the many meanings of 'generality' here is to emphasize the contrast between what Chasles is doing with the deductive axiomatic approach.

On p. 9, middle, your observation is very deep and interesting: "In other words, despite the fact that a single method was used, the ways in which it was applied were not general. The attention placed on the modalities of application of general methods, and especially their uniformity, that is, their generality, inspires many other comments in the *Aperçu historique.*" I think this is a really important point, which again might be highlighted; and it is a point that brings out the contrast with the deductive axiomatic approach.

Are we right to think that the re-representation of a geometric curve as an algebraic equation in Descartes' *Geometry* is an example (for Chasles, for you) of the imposition of uniformity in a modality of application, in the case of the method of tangents. The method of tangents applies to any (algebraic) curve, but of course first the curve must be represented as an equation. It is not enough to have a general method; one also needs a uniform modality of application; note that the way that the method of tangents is formulated, it only works for curves that are 'fitted out' (to use Nancy Cartwright's vocabulary, as I do in Ch. 2 of my book, a discussion that seems relevant here) with an equation.

Many results in modern mathematics require the imposition of a general 'modality of application' in order for the application of a general method to be possible. Thus, a re-representation of the object investigated is required, in order for methodical problem-solving to take place. My way of thinking about this situation is to say that we have an object, then superimposed on it, a re-representation (so that the original object is not replaced, but still present) and then some methodical problem-solving goes on. However, I have the sense that you (and Chasles?) think about this situation as the replacement of the object by the re-representation. I would argue that problem-solutions that require the mediation of a general 'modality of application' in order for a general method to obtain, make use of information about the original object as well as its re-representation; however, I also tend to choose examples that illustrate my ideas. Maybe we could find instances of problem-solving where (1) the object does not fade away and the information it provides is centrally important to the solution of the problem and by contrast (2) the object does fade away and provides little or no important information, while the re-representation does all the work.

Although Chasles is very appreciative of the mathematical advance that Cartesian analytic geometry gives to geometry (increasing its generality over that of classical Greek geometry), you announce that he is also critical of it. He must think that it has certain limitations that impede its problem-solving capacity, or that keep important objects or problems from coming to light. And he thinks that projective geometry is successful in certain ways that analytic geometry cannot be. But we can't find clearly stated in this essay what Chasles's objections to analytic geometry were, and so why he thinks projective geometry must be a supplement to it or in another sense a replacement. Otherwise put, could you state more clearly what Chasles thought his projective geometry could do, that analytic geometry could not do? Also, was Chasles aware of the limitations (or, of any specific limitations) of projective geometry? A few examples of results from projective geometry (just a few, and not developed at great length) might be helpful here.