Here are some comments for Evelyne Barbin, apropos her paper "Universality vs. Generality: An Interpretation of the Dispute between Descartes and Fermat over Tangents," which emerged from from my discussion about it with Emiliano Ippoliti, a postdoctoral student from the University of Rome, who is here at Penn State this summer.

The important contrast between Fermat and Descartes in this paper is given on pages 4-7. "Descartes lays claim to a universality because he has furnished a foundation for his method, namely, the knowledge of algebraic equations, at the same time that he has enclosed the field of application to his method." I understand this to mean that Descartes' claim to universality is based on (1) his identification of plane curves with algebraic equations of finite degree in two variables, (2) his method of tangents, which is set up to apply to equations of such curves, and (3) his restriction of the domain of geometry to the study of such curves, mediated by their equations. As Barbin observes here, Descartes has enclosed a field of objects / terms, and in a sense he has covered it. (In other senses, he has not: he does not really know how to handle curves with equations of degree higher than 2 or 3, or surfaces with equations involving more than two variables, or as Barbin notes transcendental plane curves. To be anachronistic, he also doesn't know how to handle the "monster" curves of the 19^{th} century; my point here is that his claims to universality are premature, and false.

Fermat's approach, as he promotes his method of maxima and minima, contrasts with that of Descartes in significant ways. (1) He does not offer a codified representation of curves, (2) his method is algebraic, but in a loose, not-formalized way, and (3) he does not try to circumscribe the domain of application of his method, but leaves it open—indeed, he expects always to find new possibilities that enlarge the domain of application. So his method is general, not universal.

We see the distinction, and think that it characterizes the approaches of the two mathematicians pretty well. However, here are some concerns.

The first has to do with the translation. We suggest a re-formulations for the Summary: "Descartes's application of a universal method takes place under certain conditions. One is conceptual, namely, the identification of a curve with a certain kind of equation; the other is procedural, namely a method formulated to apply to such equations. Moreover, he intends his method to apply to a closed domain of curves and problems, specified in algebraic terms."

Barbin then writes, "The extension of a general method by Fermat does not involve conditions." How about, "Fermat's conception of method is one of generalization rather than universalization; he does not specify algebraic conditions for the application of his method or for the characterization of objects and problems; and he views the applications of his method as an open-ended field of novel problems."

I don't think it's accurate to say that "the universal method is justified by a foundation," especially since the word 'foundation' has a technical sense in twentieth century philosophy of mathematics that is misleading here. Rather, I would say that Descartes provides a "modality of application" for his method, in Karine Chemla's sense; re-representing curves as certain kinds of equations and formulating the method of tangents to apply directly to such equations "guarantees" the success of the method, but I don't think it's helpful to call that a foundation. Arabic notation covers all the whole numbers and provides a modality of application for procedures of multiplication and long division; but we wouldn't say it provides a *foundation* for arithmetic. Peano wanted to think that his axioms (soon recast in the language of first order predicate logic, which renames the whole numbers in a notation that is not Arabic notation) provided a foundation for arithmetic, which of course they do not.

Moreover, Descartes' method was also justified by its efficacy, and was soon, for that very reason, extended to transcendental curves.

Two reflections follow from this last point. (1) Chemla invokes Chasles' perception of what he is doing as "generalization" when (in her essay, p. 9) he articulates the importance of general modalities of application for general methods. (He argues that the Greeks had a general method, but lacked a general modality of application for it, so that its application in every case required fresh ingenuity.) (2) Chasles thus would see Descartes as involved in a process of generalization. The reader may therefore see Chemla's and Barbin's paper as at odds either in the way they characterize 'generalization' or in the way they read Descartes.

The algebraization of geometrical things (here, plane curves) is a way of organizing that field of objects and problems, very different from Euclid's way of organizing it by axioms, definitions, and common notions. In Descartes' *Geometry* there is no set of axioms proposed.

Also, in the *Geometry* Descartes not only re-conceptualizes curves as equations, but in order to do that he first re-conceptualizes them as relations among straight line segments. Actually, to be precise, he re-conceptualizes the simplest geometric problems (how to construct a point under certain conditions) as problems about relations among straight line segments (in Book I) and then (in Book II) re-conceptualizes any problem about a curve as an infinite collection of problems about the simplest geometric problem: that is, he constructs curves point-wise. So curves are analyzed into points and problems involving the construction of points for Descartes only involve straight line segments; this is a radical kind of decomposition as well as linearization, which precedes, as a condition, the re-representation of curves by equations.

Chemla (in *Les Neuf chapitres* and in the essay here) usually contrasts generalization with axiomatization: *Les Neuf chapitres* is organized in an open-ended way, and exhibits a process of generalization; Euclid's *Elements* is organized as an axiomatized and therefore closed system, delineated by the construction procedure of ruler and compass. While Descartes' *Geometry* does present (or, more accurately, wishfully project) a closed system, it is not an axiomatization (like that of Euclid in one sense and Hilbert in another) and what it provides geometry with is not a foundation. Gueroult's *Descartes selon* *l'ordre des raisons* shows very well the ampliative, non-axiomatic nature of Cartesian method, and reminds us of his hostility to deductive logic.

So perhaps we need a three-fold distinction here. There is axiomatization; and then by contrast there is open-ended generalization (like Fermat's) and closed generalization (like Descartes') that achieves its closure by means of notation and method rather than axiomatization.

Attached is an interesting paper by Dirk Schlimm (McGill University) who argues that axiomatization can play a direct role in ampliative reasoning, when mathematicians are willing to "play off" abstract structure against the detail of models. That means that they must be willing to adjust the formulation of the axioms that define the abstract structure, and to adjust their conception of what the objects in the models are, how they are related, and how they might be represented. In Carlo Cellucci's view, axiomatization always freezes and closes off mathematical research; but this may be an overstatement, because it is polemical. It may be that axiomatization can play a role in generalization (in Chemla's sense); but then the axiomatization and / or its applications in models must be regarded as revisable, not fixed, and therefore amenable to mutual adjustment.

Also attached is a paper by Daniel Campos (Brooklyn College) on Charles Peirce's philosophy of mathematics, where he argues persuasively (following Peirce) that ordinary proofs in Euclid's *Elements* can be regarded as ampliative reasoning; he takes Book I, Prop. 5 as his example. This argument also suggests, in a different way, that axiomatization can play a direct role in ampliative reasoning.