Here are some comments on John Dawson's essay, "In what senses does mathematics exhibit unity, and how can such unity be accounted for?" and Carlo Cellucci's "Why Proof? What is a Proof?" which seems to me germaine to the issues discussed in Ch. 10.

Mathematics exhibits unity in the following senses:

Transferability of notions and methods: applications of mathematics within mathematics; the existence of analogies between the central features of various theories. Is this mysterious, or to be referred to a general (but so far undiscovered) theory. *(Similar structures, different objects.)*

p. 6. I like Dawson's observation that transferability is often attempted unsuccessfully; we tend to notice and remember those that are successful. I certainly remain skeptical about the discovery of some global underlying unity, not least because there are so many different candidates offering such theories. (As there are so many different theories claiming to unify quantum mechanics and relativity theory.) He mentions (pp. 6-7) that the most successful and visible unifications have been highly localized; and (p. 7) that the dichotomy between the arithmetic and geometric, the discrete and continuous, seems not to have a single bridge but rather many, none of them definitive. Finally (p. 8), theorems may only appear to remain invariant, while their terms are redefined and so either restricted or expanded.

Consilience: different theories seem to agree in their consequences when their subject matters overlap. Important mathematical results can be reached in many ways. *(Different structures, same objects.)*

The definitions of effective procedures is a good example. To me (see comments, p. 10), consilience testifies to the existence and unity of intelligible things in mathematics.

Stability over time: this perception must be nuanced by history of maths, which warns us against anachronism. Also by the historical variation in the perception of the unity / disunity of mathematics (pp. 12f.)

Intersubjective agreement.

Set theory. That set theory is a possible way to express some mathematics does not mean it is *the* privileged means of expression.

p. 14 "The aspects of unity claimed for mathematics have been exaggerated and do not serve to distinguish the mathematical enterprise to any significant degree from other fields of scientific endeavor." I agree; indeed, I think the growth of mathematical knowledge is driven by a dialectic between differentiation and unification.

The contrast between transferability and consilience can be made apropos things, or apropos discourse. Realists like to use the former; nominalists like to use the latter.

1. Transferability: a. Similar structures, different objects
b. One proof-schema, many applications

2. Consilience: a. Different structures, same object
b. Different proofs, same theorem

"Why Proof? What is a Proof?"

The question of the unity of mathematics ought to be raised along with the question of the differentiation of mathematics.

How can we account for the unity of mathematics? What does unification look like? Why is it rational to try to increase (or complete, if that's possible) the unification of mathematics? What is the point of unification? Is unification forced on us, or do we choose it?

How can we account for the differentiation of mathematics? What does differentiation look like? Why is it rational to try to increase (or maximize, if that's possible) the differentiation of mathematics? What is the point of differentiation? Is differentiation forced on us, or do we choose it?

We can say that mathematical reasoning unifies by seeking axiomatization (which is global) or by analysis (which is local).

We can say that mathematical reasoning unifies by seeking the transfer of similar structure(s?) from one kind of object to another, or by applying different structures to the same kind of object. Both transfer and consilience in this sense seem local.

We can say that mathematical reasoning differentiates when it encounters different kinds of items, with different kinds of features, presenting different kinds of problems. Since mathematics seeks problem-solving methods, it makes sense to tailor different methods to different clusters of problems about different kinds of items. Axiomatization organizes a cluster of problems. Analysis sometimes stays within clusters, sometime cuts across them.

Transfer and consilience depend on prior differentiation.

What happens when we focus on the set of questions around differentiation rather than unification? Can we maintain the pole of the global vis a vis the pole of the local?